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GAM-036022 Seat No. _________

B. Sc. (Sem. VI) Examination

March / April - 2019

Mathematics : BSCC-606B

(Analysis - III)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) Do as directed.

(2) All questions are compulsory.

(3) Each question carry equal marks.

1 (a) In a metric space prove that every open sphere is an open set. **7**

OR

Let X be a metric space then prove that

(1) Any intersection of closed set is closed.

(2) Finite union of closed sets is closed.

(b) Let $X = R^2$ and $d : R^2 \times R^2 \to R$ be defined as **7**

$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ where $x = (x_1, x_2)$

$y = (y_1, y_2) \in R^2$ then prove that d is a metric on R^2.

OR

Let X be a metric pace, A subset F of X is closed iff it's complement F' is open.

2 (a) Prove that if A is closed subset of a compact metric space (X, d) then (A, d) is compact. **7**

OR

Let X and Y be metrix space and f is a mapping of X in to Y, then prove that f is continuous iff $f^{-1}(G)$ is open in X, whenever G is open in Y.

(b) Prove that continuous image of a compact metric space is compact. **7**

OR

Let X and Y be metric spaces and $f: X \to Y$ then prove that f is continuous at x_0 iff $f(x_n) \to f(x_o)$ whenever $x_n \to x_o$.

3 (a) Let $\{f_n\}$ be a sequence of continuous function on $[a,b]$ and suppose that $f_n \to f$ uniformly on $[a,b]$ then prove that **7**

$$\lim_{n\to\infty} \int_a^b f_n(x)\,dx = \int_a^b \lim_{n\to\infty} f_n(x)\,dx.$$

OR

Suppose $\lim\limits_{n\to\infty} f_n(x) = f(x)$ $\forall x \in E$ and let

$M_n = \sup\limits_{x\in E} \left|f_n\infty - f(x)\right|$ then $f_n \to f$ uniformaly on E iff

$\lim\limits_{n\to\infty} M_n = 0$.

(b) Let $f_n(x) = |x|^{1+\frac{1}{n}}$ $\forall x \in [-1, 1]$ show that **7**

(1) $f_n \in D[-1, 1]$

(2) $f_n(x) \Rightarrow f(x) = |x|$ uniformly on $[-1, 1]$

(3) f is not diffrentiable

OR

Prove that $f_n(x) = n^2 x^n (1-x)$ $\forall x \in [0, 1]$ converges point wise but not uniformaly to a function which is continuous in $[0, 1]$.

4 (a) State and prove Weierstrass approximation theorem. **7**

OR

State and prove Abel's limit theorem.

(b) For every $x \in R$ and $n \geq \theta$ prove that **7**

(1) $$\sum_{k=\theta}^{n} (nx-k)^2 \binom{n}{k} x^k (1-x)^{n-k} = nx(1-x)$$

(2) $$\sum_{k=\theta}^{n} \binom{n}{k} x^k (1-x)^{n-k} = 1$$

OR

Show that the function

$$f(x) = \left.\begin{matrix} e^{\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{matrix}\right\}$$ has derivatives of all orders at all

$x \neq 0$ but doesnot have a Taylor's theorem.

5 Answer in short : (any seven) **14**

(1) Define : Open set and closed set.

(2) Define : Convergence and completeness.

(3) Define : Continuty and compactness.

(4) Define : Pointwise and uniformly convergence.

(5) Define : A derive set. Give one example.

(6) Define : A connected set. Give an example of connected set.

(7) If $f_n(x) = \frac{1}{1+nx}(x \geq 0)$ then check continuity of $f_n(x)$.

(8) Define : Metric space.

(9) Explain Taylor's series.