* G A O - 0 3 6 0 2 4 *

GAO-036024 Seat No._____

B. Sc. (Sem. VI) Examination

March / April - 2019

BSCC-606D : Graph Theory : MAT-310

Time : 3 Hours] [Total Marks : 70

Instructions:

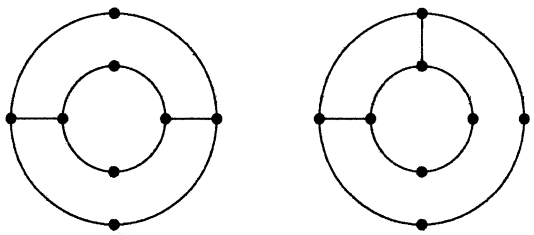
1. All the questions are compulsory.
2. Figure to the right side indicate marks.
3. Each sub questions carries equal weightage.
4. All notations are standard.

1. (a) Define a graph. Prove that, in any graph G there is an even number of odd vertices. **7**

OR

Define path. Prove that, given any two vertices u and v of a graph G, every $u - v$ walk contains a $u - v$ path.

(b) Define isomorphism of graphs. Are the following two graphs isomorphic? Why? **7**



OR

(b) Define degree of a vertex. Let G be a graph with n vertices, t of which have degree k and the others have degree $k + 1$. Prove that $t = (k + l)n - 2e$, where e is the number of edges in G.

2. (a) Prove that if T is a tree with n vertices then it has precisely $n - 1$ edges. **7**

OR

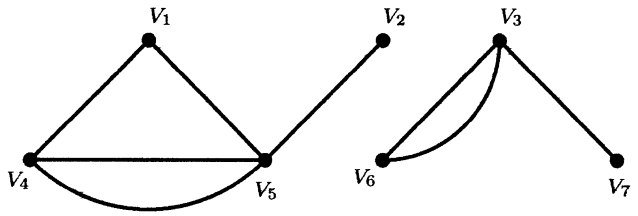
(a) Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G.

(b) Draw the graph having the following matrix as its adjacency matrix. **7**

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

OR

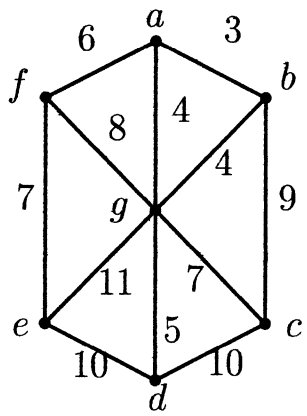
(b) Using fusion method show that the following graph is not connected.



3. (a) Prove that a graph G is connected if and only if it has a spanning tree. **7**

OR

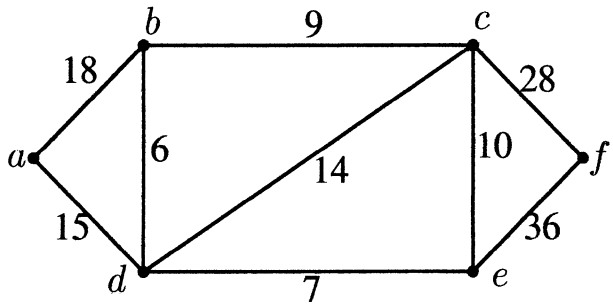
Find a minimal spanning tree of the following graph using Kruskal's algorithm.



(b) Define cut vertex. Prove that for a graph G with n vertices, where $n \geq 2$, there are at least two vertices which are not cut vertices. **7**

OR

(b) Use Dijkstra's algorithm to find the shortest path from vertex a to vertex f for the following graph.



4. (a) Define Euler graph. Prove that for a connected Euler graph G the degree of every vertex of G is even. **7**

OR

Define Hamiltonian Graph. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.

(b) Prove that a connected graph G has an Euler trail if and only if it has at most two odd vertices. **7**

OR

Describe Königsber seven bridge problem.

5. Answer the following in short: (any seven) **14**

(1) Draw K_5, the complete graph with 5 vertices.

(2) Define Bipartite graph.

(3) Define Subgraph of a graph.

(4) Give example of 3-regular graph.

(5) For a acyclic graph G with n vertices and k connected components, G has _____ edges.

(6) Define bridge in a graph.

(7) Define spanning tree.

(8) The total number of edges in a complete graph G with 6 vertices is _____.

(9) True or False (Justify your answer): Every tree is a bipartite graph.