GAN-036023 **Seat No.** _________

B. Sc. (Sem. VI) Examination

March / April - 2019

Mathematics : BSCC-606C

(Abstract Algebra - II)

Time : **3** Hours] [Total Marks : **70**

Instructions :
(1) Do as directed.
(2) Total five questions in this question paper.
(3) Right side of number indicate full marks of question/subquestion.

1 (a) Define ring and division ring. **7**

In a ring $(R,+,\bullet)$ for $a,b\in R$ prove that

(1) $a\cdot 0=0\cdot a=0$

(2) $a(-b)=(-a)b=-(ab)$

(3) $(-a)(-b)=ab$

where 0 is the zero element of ring R.

OR

Show that the set $Q\left(\sqrt{2}\right)=\left\{a+b\sqrt{2}\,/\,a,b\in Q\right\}$ is a ring under addition and multiplication.

(b) Prove that every finite integral domain is a field. **7**

OR

Define characteristic of a ring.

Prove that the characteristic of an integral domain is either prime number or zero.

2 (a) Define Subring. **7**

Let R be a ring and U be a non-empty subset of R then U is a subring of R iff

(i) $\forall\, a, b \in U \quad a - b \in U$

(ii) $\forall\, a, b \in U \quad a \cdot b \in U$

OR

Define Principal Ideal.

Prive that a field has no proper ideal.

(b) State and prove fundamental theorem on homomorphism of ring. **7**

OR

Define Kernel of a ring homomorphism.

If f is a homomorphism of a ring R with Kernel K, then prove that K is an ideal of ring R.

3 (a) Define degree of a polynomial in $D(x)$. If $f, g \in D(x)$ are non-zero polynomials then prove that **7**

$\deg(f \cdot g) = \deg f + \deg g$.

OR

If D' is the set of all constant polynomials of $D(x)$ then prove that $D \cong D'$.

(b) State and prove Division algorithm for polynomials. **7**

OR

State and prove Remainder theorem.

Also find all zeores of

$x^4 + 3x^3 + 2x + 4 \in Z_5(x)$ in Z_5.

4 (a) Prove that an ideal S of a commutative ring R with unity is maximal iff the quotient ring $R \mid S$ is a field. **7**

OR

A non-empty subset U of a field F is a subfield of F iff

(i) $\forall x, y \in U, \quad x - y \in U$

(ii) $\forall x, y \in U, \quad xy^{-1} \in U$

(b) An ideal S of the ring of integer I is maximal iff S is generated by some prime integer. **7**

OR

If D is an integral domain $a, b \in D$,

(i) for $(m, n) = 1, \; a^m = b^m \;\; a^n = b^n$ then prove $a = b$

(ii) If P is the characteristic of an integral domain R, then prove that for $a, b \in R \;\; (a+b)^P = a^P + b^P$.

5 Answer in short : (any seven) **14**

(1) If R is boolean ring then prove that $a + a = 0 \quad \forall a \in R$.

(2) Define : Integral domain and give one example.

(3) Define : Field.

(4) Define : Left and Right ideals on R.

(5) Define : G.C.D. of two polynomials.

(6) Give an example of left ideal which is not right ideal.

(7) Prove that boolean ring is a commutative ring.

(8) Define : Principal ideal.

(9) Define : Homomorphism ring.