



G-035021

Seat No. _____

B. Sc. (Sem. V) Examination

April/May - 2019

**Mathematics - BSCC-506-A
(Analysis-I)**

Time : 3 Hours]

[Total Marks : 70

- Instructions :** (1) All the questions are compulsory and carry 14 marks.
(2) Notations are as usual.
(3) Figures to the right side indicate marks.

- 1 (a) Prove that countable union of countable sets is again countable. 7

OR

- (a) Define sup. and inf. of a non-empty set s . Let s be a non-empty subset which is bounded above, let $a \in R$ any no. prove that

$$\sup(a + s) = a + \sup s$$

- (b) Prove that there exists a positive real no x such that $x^2 = 2$. 7

OR

- (b) State and prove rational density theorem.

- 2 (a) Prove that a bounded and monotonically increasing sequence is converges.

OR

- (a) Prove that a sequence is Cauchy iff it is convergent.
(b) State and prove Bolzano-Weierstrass theorem.

OR

- (b) State and prove nested interval theorem.

- 3 (a) State and prove De Moivre's theorems. Hence solve the equation $x^3 + 1 = 0$.

OR

- (a) State the triangle inequality for the complex number.
Hence show that

$$\|z_1\| - \|z_2\| \leq |z_1 - z_2|$$

- (b) If z_1 and z_2 are two complex nos. then such that

$$|z_1 + z_2| = |z_1 - z_2| \text{ then prove that } \arg z_1 - \arg z_2 = \frac{\pi}{2}$$

OR

- (b) If $\sin(\alpha + i\beta) = x + iy$ then prove that

$$(i) \quad \frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$$

$$(ii) \quad \frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$$

- 4 (a) Prove that the necessary conditions for a function $f(z) = u + iv$ to be analytic at all points in a region R are.

- (i) $ux = vy$
(ii) $uy = -vx$.

OR

- (a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is any analytic function of $z = x + iy$, find $f(z)$ in term of z .
- (b) If $f(z) = u(r, \theta) + iv(r, \theta)$ be a analytic function and $u = -r^3 \sin 3\theta$ then find a function $v(r, \theta)$ and also express the function $f(z)$ in terms of z .

OR

- (b) Show that the function $w = \sin z$ satisfy the Cauchy-Rieman equations. Find its derivative.

5 Answer in short : (any seven)

14

- (1) If $z_1 = 2 + 3i$, $z_2 = 4 - i$ then find $z_1 z_2$.
 - (2) Find the modulus and principal argument of the complex number $3 + 4i$.
 - (3) Define injective and subjective function with example.
 - (4) Give an example of bounded sequence which is not convergent.
 - (5) If $x + iy = \frac{1}{a + bi}$ prove that $(x^2 + y^2)(a^2 + b^2) = 1$.
 - (6) Prove that $\sin(ix) = i \sinh(x)$.
 - (7) State at least four properties of complex number.
 - (8) Define :
 - (i) Harmonic conjugate of a function
 - (ii) Linear transformation.
 - (9) Define trigonometric and hyperbolic functions for the complex variable.
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