GL-036021 Seat No. _________

B. Sc. (Sem. VI) Examination

March / April - 2019

BSCC606A : Analysis-II

(MAT-307)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Figure to the right side indicate marks.
(3) Each sub question carries equal weightage.
(4) All notations are standard.

1 (a) Let g be continuous on $[a,b]$ and f has a derivative which is continuous and never changes sign on $[a,b]$, then for some $c \in [a,b]$ prove that **7**

$$\int_a^b f(x)g(x)dx = f(a) \int_a^c g(x)dx + f(b) \int_c^b g(x)dx$$

OR

If P_2 is a refinement of the partition P_1 of $[a,b]$, then prove that $U[f,P_2] \le U[f,P_1]$ and $L[f,P_2] \ge L[f,P_1]$.

(b) If P_1 and P_2 are two partitions of $[a,b]$, then prove that $U[f,P_1] \ge L[f,P_2]$. **7**

OR

Let $f(x) = \frac{x^2}{3}$. For each $n \in \mathbb{N}$, σ_n be the partition $\left\{0, \frac{1}{n}, \frac{2}{n}, \ldots\ldots, \frac{n-1}{n}, 1\right\}$, then compute $\lim\limits_{n\to\infty} U[f;\sigma_n]$ and $\lim\limits_{n\to\infty} L[f;\sigma_n]$.

2 (a) State and prove Cauchy's condensation test and hence prove that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$ converges. **7**

OR

Let $\sum a_n$ be a divergent series of positive numbers. Prove that a sequence $\{\epsilon_n\}$ of positive number exists such that $\{\epsilon_n\}$ converges to zero but $\sum_{n=1}^{\infty} \epsilon_n a_n$ diverges.

(b) State and prove the limit form of the comparison test for the convergence of the series. Discuss the convergence of $\lim_{n \to 1} \frac{1+n}{2+3n^{3/2}}$. **7**

OR

Prove that the absolutely convergent series is convergent. Is converse true ? Justify your answer.

3 (a) If $\sum a_n$ is absolutely convergent then prove that any rearragement of $\sum a_n$ has the same sum. **7**

OR

If $\sum a_n z_0^n$ is convergent, then prove that $\sum a_n z^n$ is absolutely convergent for $|z| < |z_0|$. Also discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{z^n}{(n+1)^{\alpha}} (\alpha \in \mathbb{R})$.

(b) Find the radius of the convergence and interval of convergence of the following power series : **7**

(i) $\sum_{n=1}^{\infty} \frac{n^2 (x-2)^n}{2^n}$

(ii) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

OR

If the series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ converges absolutely to A and B respectively then prove that their Cauchy product $\sum_{n=0}^{\infty} C_n$ is convergent. If C is the sum of the Cauchy product then $C = AB$.

4 (a) Show that for $-1 < x < 1$, **7**

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots + \frac{(-1)^{n-1} x^n}{n} + \ldots\ldots..$$

OR

Obtain Maclaurin series expansion of $\sin x$ for $-\infty < x < \infty$.

(b) Obtain the power series solution of the differential equation $y'' + y = 0$ with the condition $y(0) = 0, y'(0) = 1$. **7**

OR

State and prove Binomial series theorem.

5 Answer the following in short : (any seven) **14**

(1) For $f(x) = \sin 2x$ on $[0, \pi]$, find $U[f, P]$ for the partition $P = \left\{0, \frac{\pi}{2}, \pi\right\}$.

(2) Find the primitive F for $f = 4\sin x + 3e^x$.

(3) Discuss the convergence of the series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots\ldots$

(4) Evaluate : $\int_0^4 [x]\, dx$.

(5) Discuss the absolute convergence of $\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n}$.

(6) Find the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{n(x-4)^n}{(n+3)4^n}.$$

(7) Define Cauchy product of two series.

(8) Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^3 x^n}{n!}.$$

(9) State Maclaurin's theorem with Cauchy form of reminder.